Cormack's implicit scheme made in the original paper, although algebraically correct, does not seem to be the most suitable. Due to its resemblance to the explicit scheme, the following parameters are proposed: $\xi = \frac{1}{2}$, θ is related to the parameter λ introduced in the MacCormack scheme [Ref. 3, Eq. (5)] by $\theta = \lambda / |a|$. With MacCormack's choice,

$$\lambda = \max\left\{ |a| - \frac{1}{2\sigma},\ 0 \right\}$$

This yields

$$\theta = \max\left\{ 1 - \frac{1}{2\sigma |a|},\ 0 \right\}$$

This choice of parameters appears more convenient in the sense that when the implicit option is not used in MacCormack's scheme ($\lambda = 0$), the parameter θ is zero.

Finally, since according to the stability analysis in Ref. 1, the optimum choice for the dissipation of numerical modes is, in this case,

$$\theta = \frac{\sqrt{2}}{2} - \frac{1}{2\sigma |a|}$$

the optimum choice for a scheme taking advantage of the explicit option whenever possible from stability considerations would be

$$\theta = \max\left\{ \frac{\sqrt{2}}{2} - \frac{1}{2\sigma |a|},\ 0 \right\}$$

References

[1]Casier, F., Deconinck, H., and Hirsch, Ch., "A Class of Bidiagonal Schemes for Solving the Euler Equations," *AIAA Journal*, Vol. 22, Nov. 1984, pp. 1556-1563.

[2]MacCormack, R. W., "The Effect of Viscosity in Hypervelocity Impact Cratering," AIAA Paper 69-354, April 1969.

[3]MacCormack, R. W., "A Numerical Method for Solving the Equations of Compressible Viscous Flow," *AIAA Journal*, Vol. 20, Sept. 1982, pp. 1275-1281.

Reply by Authors to G. Degrez

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THE authors[1] generally agree with this Comment. Indeed, the central bidiagonal scheme (CBS) contains two parameters (θ and ξ), whereas the McCormack implicit scheme contains only one parameter (λ). This allows the free choice of one of the CBS parameters (e.g., ξ) for the identification of both schemes.

The choice of $\xi = \frac{1}{2}$ made by Degrez is indeed preferable if one wishes to compare or to switch to the explicit McCormack scheme obtained for $\xi = \frac{1}{2}$, $\theta = 0$. In particular, this choice allows the traditional interpretation of the space derivatives in the McCormack implicit scheme as one-sided downwind in the U-sweep predictor step [discretized at $i + \frac{1}{2} - \xi = i$ using cell $(i, i+1)$] and one-sided upwind in the L-sweep corrector step [discretized at $i + \frac{1}{2} + \xi = i + 1$ using cell $(i, i+1)$]. As a result, each step is first-order accurate in space. Note that the McCormack scheme as described in Ref. 2 of the Comment is a U-L scheme with sweeps from right to left in the predictor and left to right in the corrector step, explaining the reversed sign of ξ compared to the L-U scheme described in our paper.[1]

In the context of the paper, however, the choice $\xi = 0$ leading to $\theta = 1$ has some attractive aspects: The type of calculations made by the authors uses a constant local CFL number in each meshpoint, equal to the value specified in the data file. Hence, the explicit stability condition is never satisfied in any point of the mesh and application of the McCormack implicit scheme would never switch to the explicit scheme in this case. Thus, the choice of $\theta = 1$ is independent of the CFL number, as opposed to the choice made in the Comment.

Further, the choice $\xi = 0$, $\theta = 1$ shows the close resemblance with the optimal fully implicit scheme used in the numerical tests in the paper and determined by the choice $\xi = 0$, $\theta = \sqrt{2}/2 = \pm 0.7$.

Finally, the choice $\xi = 0$, $\theta = 1$ shows that the McCormack implicit scheme can be interpreted as resulting from a central second-order discretization in the point $i + \frac{1}{2}$ in both predictor and corrector steps (if no use is made of the explicit option). This central "box" interpretation of the McCormack implicit scheme strongly differs from the usual interpretation and can have important consequences, e.g., in the presence of a source term as in the quasi-one-dimensional Euler equations. The source term for the scheme $\xi = \frac{1}{2}$ would be discretized at i in the predictor step and at $i + 1$ in the corrector step, precisely as in the traditional explicit McCormack scheme. For the scheme $\xi = 0$, however, the source term would be discretized at $i + \frac{1}{2}$ in both the predictor and corrector step,[1] which is obtained by taking the average over the values at i and $i + 1$. On the other hand, with $\xi = 0$, each step is second-order accurate in space at $i + \frac{1}{2}$.

Again, the identification proposed by Degrez is more in line with the traditional McCormack approach.

References

[1]Casier, F., Deconinck, H., and Hirsch, Ch., "A Class of Bidiagonal Schemes for Solving the Euler Equations," *AIAA Journal*, Vol. 22, Nov. 1984, pp. 1556-1563.

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Comment on "Application of the Generalized Inverse in Structural System Identification"

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THE use of the Moore-Penrose generalized inverse underlies much of the current work in structural dynamics. Whereas Chen and Fuh[1] and Berman[2,3] use this inverse explicitly (and correctly) in model adjustment using identified modes, it is often also used implicitly in the identification algorithms that provide the data for model adjustment. In this application it is not always clear that the analyst is aware of the limitations of the method. The essential difference is that Chen and Fuh[1] are able to assume in their analysis that "the measured modal matrix Φ ($n \times m$) is rectangular with full column rank m." In the identification stage, however, the rank of the modal matrix corresponds to the number of identifiable modes in the test data. Under many test conditions the decision as to the number of identifiable modes is not clear-cut and depends on the subjective judgment of the analyst.

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